

No warm up today, draw a smiley instead.

Sit near where your team was.

HW Quiz first, no calculator, about 5-10 min.

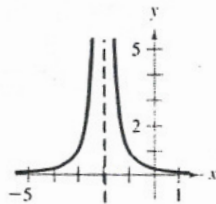
Turn in your HW quiz and take a tan limit review classwork worksheet. Work quietly.

Homework questions and limit review will follow.

HW Questions: p. 82

In Exercises 1 and 2, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches -2 from the left and from the right.

1. $f(x) = \frac{1}{(x+2)^2}$



In Exercises 3–6, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches -3 from the left and from the right.

5. $f(x) = \frac{x^3}{x^2 - 9}$

In Exercises 9–18, find the vertical asymptotes (if any) of the given function.

9. $f(x) = \frac{1}{x^2}$

13. $f(x) = \frac{x^3}{x^2 - 4}$

17. $f(x) = \frac{x}{x^2 + x - 2}$

In Exercises 19–22, determine whether the given function has a vertical asymptote or a removable discontinuity at $x = -1$.

21. $f(x) = \frac{x^2 + 1}{x + 1}$

In Exercises 23–32, find the indicated limit.

25. $\lim_{x \rightarrow 4^+} \frac{x^2}{x^2 - 16}$

29. $\lim_{x \rightarrow 1} \frac{x^2 - x}{(x^2 + 1)(x - 1)}$

$\lim_{x \rightarrow 1} \frac{x}{x^2 + 1}$

In Exercises 33–38, find the indicated limit (if it exists), given that

$$f(x) = \frac{1}{(x-4)^2} \quad \text{and} \quad g(x) = x^2 - 5x.$$

33. $\lim_{x \rightarrow 4} f(x)$

37. $\lim_{x \rightarrow 4} \left[\frac{f(x)}{g(x)} \right]$

$$\begin{aligned} &= \frac{\lim_{x \rightarrow 4} f(x)}{\lim_{x \rightarrow 4} g(x)} \\ &= \frac{\frac{1}{0} \rightarrow \infty}{-4} \\ &= -\infty \end{aligned}$$

41. A 25-foot ladder is leaning against a house, as shown in the figure. If the base of the ladder is pulled away from the house at a rate of 2 feet per second, the top will move down the wall at a rate of

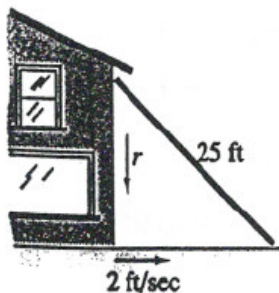
$$r = \frac{2x}{\sqrt{625 - x^2}} \text{ ft/sec.}$$

$$\lim_{x \rightarrow 25^-} \frac{2x}{\sqrt{625 - x^2}}$$

- (a) Find the rate when x is 7 feet.
 (b) Find the rate when x is 15 feet.
 (c) Find the limit of r as $x \rightarrow 25^-$.

$$\frac{50}{0}$$

$$\infty$$



Limit Review

Use reasoning to determine the limit.

$$1. \lim_{x \rightarrow 0} \frac{x}{3}$$

$$2. \lim_{x \rightarrow 0^+} \frac{3}{x}$$

$$\lim_{x \rightarrow 0^-} \frac{3}{x}$$

Limit Review

Use reasoning to determine the limit.

$$1. \lim_{x \rightarrow 0} \frac{x}{3} = 0$$

$$2. \lim_{x \rightarrow 0^+} \frac{3}{x} \quad \frac{3}{0^+} = \infty$$

$0^+ \leftarrow \text{sm pos. \#}$

$$\lim_{x \rightarrow 0^-} \frac{3}{x} \quad \frac{3}{0^-} \leftarrow \text{sm neg. \#} = -\infty$$

$$\lim_{x \rightarrow 0} \frac{3}{x} \quad \text{DNE vertical asymptote @ } x=0$$

$$3. \bullet \lim_{x \rightarrow 2} \frac{-5}{x-2}$$

$$\lim_{x \rightarrow 2^-} \frac{-5}{0^-} = \infty$$

$$\lim_{x \rightarrow 2^+} \frac{-5}{0^+} = -\infty$$

$$\text{so } \lim_{x \rightarrow 2} \frac{-5}{x-2} \text{ DNE}$$

$$4. \bullet \lim_{x \rightarrow c} f(x) = \infty$$

$$\lim_{x \rightarrow c} g(x) = 2$$

$$\text{find } \lim_{x \rightarrow c} \frac{f(x)}{g(x)}$$

$$\text{and } \lim_{x \rightarrow c} \frac{g(x)}{f(x)}$$

$$\frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)}$$

$$\frac{\infty}{2}$$

$$\infty$$

$$\frac{2}{\infty}$$

$$0$$

Find any vertical asymptotes. Verify with the limit definition of a vertical asymptote.

$$f(x) = \frac{x^2 - 2}{x^2 - x - 2}$$

$$f(x) = \frac{x^2 - 2}{(x - 2)(x + 1)}$$

Vertical
asympt @ $x=2$
and $x=-1$

$$\lim_{x \rightarrow 2} f(x)$$

$$= \frac{2}{0}$$

$$= \infty$$

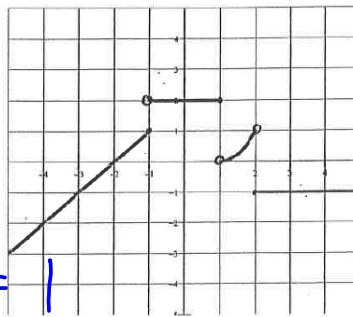
$$\lim_{x \rightarrow -1} f(x)$$

$$= \frac{-1}{0}$$

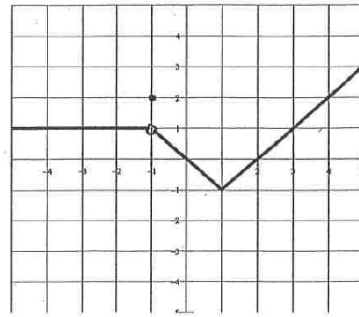
$$= -\infty$$

← justification

Classwork:



Graph of f



Graph of g

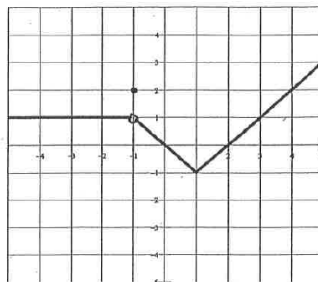
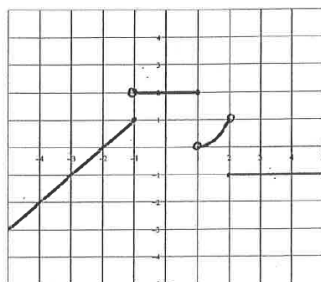
$$1. \lim_{x \rightarrow -1^-} f(x) = 1$$

$$2. \lim_{x \rightarrow -1^+} f(x) = 2$$

$$3. \lim_{x \rightarrow -1} f(x) \text{ DNE}$$

$$4. \lim_{x \rightarrow -1^-} g(x) = 1$$

$$5. \lim_{x \rightarrow -1^+} g(x) = 1$$



$$6. \lim_{x \rightarrow -1} g(x) = 1$$

Graph of f

Graph of g

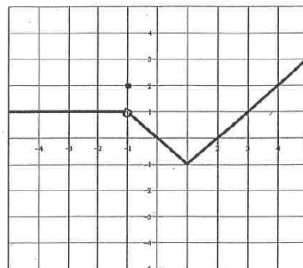
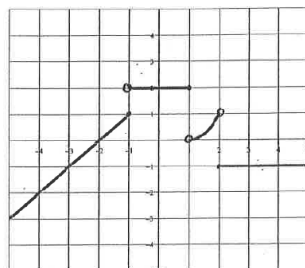
$$7. \lim_{x \rightarrow 1} f(x) \text{ DNE b/c } \lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$$

$$8. \lim_{x \rightarrow 1} g(x) = -1 \quad 2 \neq 0$$

$$9. \lim_{x \rightarrow -1} (f(x)g(x)) \rightarrow \text{DNE b/c } \left[\lim_{x \rightarrow -1} f(x) \right] \left[\lim_{x \rightarrow -1} g(x) \right]$$

$$10. \lim_{x \rightarrow 0} (2f(x) + 3g(x)) \quad (\text{DNE } x = -1)$$

4



$$11. \lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \frac{2}{0} = \infty \text{ DNE}$$

Graph of f

Graph of g

$$12. \lim_{x \rightarrow 0} \frac{g(x)}{f(x)} = \frac{0}{2} = 0$$

$$13. \lim_{x \rightarrow -2} g(f(x)) = 0 \quad \begin{cases} \textcircled{1} \lim_{x \rightarrow -2} f(x) = 0 \\ \textcircled{2} \lim_{x \rightarrow 0} g(x) = 0 \end{cases}$$

$$14. \lim_{x \rightarrow -1} f(g(x)) \rightarrow \lim_{x \rightarrow -1} g(x) = 1$$

$$\lim_{x \rightarrow 1} f(x) = \text{DNE}$$

HW: p. 75, # 6, 11, 12, 16, 19,
20, 39, 43

and p. 82, # 23, 35, 36, 38

Friday: Test Ch 2